South Carolina College- and Career-Ready Standards for Mathematics

6th Grade Support Document

South Carolina Department of Education
Office of Standards and Learning
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South Carolina College- and Career-Ready Standards for Mathematics
Grade 6 Overview

The Table of Contents below arranges the *South Carolina College- and Career-Ready Standards for Mathematics* for middle school into *Course Coversheets* and *Units*.

- Each middle school *Course Coversheet* organizes the middle school course standards into possible instructional units and provides links to specific middle school course *Units*.
- Each middle school course *Unit* contains:
  - Clarifying notes related to the standards within the unit
  - New academic vocabulary in the unit
  - Prior and subsequent knowledge related to the unit
  - Description of the relationship between the standards in the unit
  - Potential instructional strategies and lessons
  - Resources for the unit
  - Sample formative assessment tasks and questions

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Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7
Number System	Ratios and Rates	Graphing and Rational Numbers	Expressions	Equations	Statistics	Geometry
Standards	**Standards**	**Standards**	**Standards**	**Standards**	**Standards**	**Standards**
6.NS.1 6.NS.2 6.NS.3 6.NS.4 6.NS.9	6.RP.1 6.RP.2 6.RP.3	6.NS.5 6.NS.6 6.NS.7 6.NS.8	6.EEI.1 6.EEI.2 6.EEI.3 6.EEI.4	6.EEI.5 6.EEI.6 6.EEI.7 6.EEI.8 6.EEI.9	6.DS.1 6.DS.2 6.DS.3 6.DS.4 6.DS.5	6.GM.1 6.GM.2 6.GM.3 6.GM.4

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Mathematical Process Standards: The South Carolina College- and Career-Ready (SCCCR) Mathematical Process Standards demonstrate the ways in which students develop conceptual understanding of mathematical content and apply mathematical skills. As a result, the SCCCR Mathematical Process Standards should be integrated within the SCCCR Content Standards for Mathematics for each grade level and course. Since the process standards drive the pedagogical component of teaching and serve as the means by which students should demonstrate understanding of the content standards, the process standards must be incorporated as an integral part of overall student expectations when assessing content understanding.

1. Make sense of problems and persevere in solving them. a. Relate a problem to prior knowledge. b. Recognize there may be multiple entry points to a problem and more than one path to a solution. c. Analyze what is given, what is not given, what is being asked, and what strategies are needed, and make an initial attempt to solve a problem. d. Evaluate the success of an approach to solve a problem and refine it if necessary.	**5. Use a variety of mathematical tools effectively and strategically.** a. Select and use appropriate tools when solving a mathematical problem. b. Use technological tools and other external mathematical resources to explore and deepen understanding of concepts.
2. Reason both contextually and abstractly. a. Make sense of quantities and their relationships in mathematical and real-world situations. b. Describe a given situation using multiple mathematical representations. c. Translate among multiple mathematical representations and compare the meanings each representation conveys about the situation. d. Connect the meaning of mathematical operations to the context of a given situation.	**6. Communicate mathematically and approach mathematical situations with precision.** a. Express numerical answers with the degree of precision appropriate for the context of a situation. b. Represent numbers in an appropriate form according to the context of the situation. c. Use appropriate and precise mathematical language. d. Use appropriate units, scales, and labels.
3. Use critical thinking skills to justify mathematical reasoning and critique the reasoning of others. a. Construct and justify a solution to a problem. b. Compare and discuss the validity of various reasoning strategies. c. Make conjectures and explore their validity. d. Reflect on and provide thoughtful responses to the reasoning of others.	**7. Identify and utilize structure and patterns.** a. Recognize complex mathematical objects as being composed of more than one simple object. b. Recognize mathematical repetition in order to make generalizations. c. Look for structures to interpret meaning and develop solution strategies.
4. Connect mathematical ideas and real-world situations through modeling. a. Identify relevant quantities and develop a model to describe their relationships. b. Interpret mathematical models in the context of the situation. c. Make assumptions and estimates to simplify complicated situations. d. Evaluate the reasonableness of a model and refine if necessary.	

Unit Title
Number System
Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*
6.NS.1 Compute and represent quotients of positive fractions using a variety of procedures (e.g., visual models, equations, and real-world situations). ○ Divide positive fractions by fractions using visual models and equations. ○ Solve real-world problems using division of fractions. **6.NS.2** Fluently divide multi-digit whole numbers using a standard algorithmic approach. ○ Divide whole numbers by whole numbers Convert remainders to fractional parts in simplest form and decimal notation **6.NS.3** Fluently add, subtract, multiply and divide multi-digit decimal numbers using a standard algorithmic approach. ○ Perform all operations with decimal notation Modeling equivalent numerical expressions to support the understanding of division of decimal numbers **6.NS.4** Find common factors and multiples using two whole numbers. ○ Compute the greatest common factor (GCF) of two numbers both less than or equal to 100. ○ Compute the least common multiple (LCM) of two numbers both less than or equal to 12. ○ Express sums of two whole numbers, each less than or equal to 100, using the distributive property to factor out a common factor of the original addends. ○ Understand that greatest common factor and least common multiple are ways to discuss number relationships in multiplication and division. ○ Understand the process of prime factorization. ○ Understand the distributive property using sums and its use in adding numbers 1-100 with a common factor. ○ Use LCM and GCF to teach fluency for adding and subtracting of fractions using a standard algorithmic approach. **6.NS.9** Investigate and translate among multiple representations of rational numbers (fractions, decimal numbers, percentages). Fractions should be limited to those with denominators of 2, 3, 4, 5, 8, 10, and 100. ○ Recognize ⅛ as half of ¼ to assist with conversions within all representations. ○ Understand that fractions with a denominator of 3 will generate a repeating decimal (limit repeating decimals to fractions with a denominator of 3).

New Academic Vocabulary for This Unit
● Reciprocal ● Inverse ● Greatest common factor ● Least common multiple ● Prime factorization ● Distributive property ● Rational number

Prior Knowledge Required for this Unit
● Multiplication facts (3.ATO.1, 3.ATO.3) ● Understand the relationship between multiplication and division (5.NSF.3) ● Understand parts of a fraction (3.NSF.1) ● Divide up to four-digit dividends by two-digit divisors (5.NSBT.6) ● Add, subtract, multiply, and divide decimal numbers to hundredths using concrete area models and drawings (5.NSBT.7) ● Firm conceptual understanding of place value (3.NSBT.1, 4.NSBT.1, 5.NSBT.1)
Subsequent Knowledge Related to this Unit
This unit will end direct instruction for operations with whole numbers, fractions, and decimals. To ensure readiness for work with integers in Grade 7, students must be computationally fluent with these operations. In Grade 8, students will be solving multi-step equations where the computational skills will be secondary skills in an algebraic approach. Students will begin multiple representations of rational numbers with limited denominators in Grade 6. That knowledge in Grades 7 and 8 will be extended to include all denominators and repeated decimals in Grade 8. The information taught in this unit will also prepare students for ratios and rates including work with greatest common factor and least common multiple for simplifying rates. This knowledge will be extended in Grade 8 to include work with functions including linear functions where students will analyze slope as the constant rate of change.
Relationship Among Standards in this Unit
Standards in this unit are all necessary to develop computational skills necessary for work with positive rational numbers.

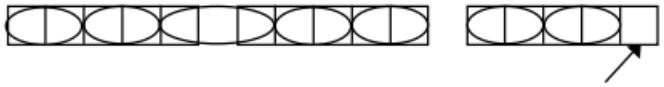
Potential Instructional Strategies/Lessons

- Visual models - Multiple representations of visual models should be used to show multiplication and division of fractions.

Example 1:

Students understand that a division problem such as $3 \div \frac{2}{5}$ is asking, "how many $\frac{2}{5}$ are in 3?" One possible visual model would begin with three whole and divide each into fifths. There are 7 groups of two-fifths in the three wholes. However, one-fifth remains. Since one-fifth is half of a two-fifths group, there is a remainder of $\frac{1}{2}$.

Therefore, $3 \div \frac{2}{5} = 7\frac{1}{2}$, meaning there are $7\frac{1}{2}$ groups of two-fifths. Students interpret the solution, explaining how division by fifths can result in an answer with halves.



This section represents one-half of two-fifths

Students also write contextual problems for fraction division problems. For example, the problem, $\frac{2}{3} \div \frac{1}{6}$ can be illustrated with the following word problem:

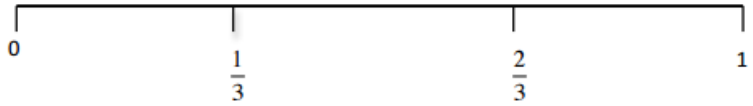
Source: NC DPI 6th Grade Mathematics Unpacked Contents

Example 2:

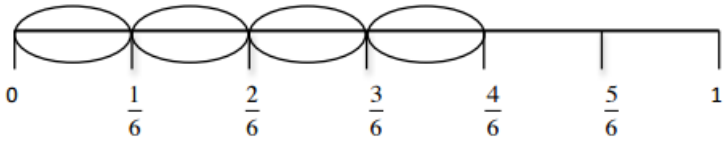
Susan has $\frac{2}{3}$ of an hour left to make cards. It takes her about $\frac{1}{6}$ of an hour to make each card. About how many can she make?

This problem can be modeled using a number line.

a. Start with a number line divided into thirds.



b. The problem wants to know how many sixths are in two-thirds. Divide each third in half to create sixths.

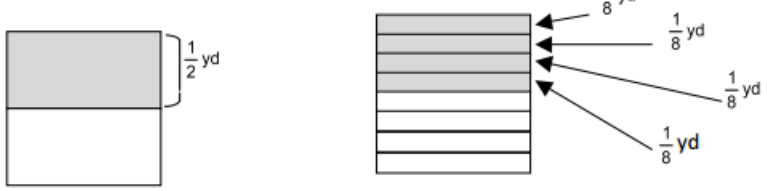


c. Each circled part represents $\frac{1}{6}$. There are four sixths in two-thirds; therefore, Susan can make 4 cards.

Source: NC DPI 6th Grade Mathematics Unpacked Contents

Example 3:

Michael has $\frac{1}{2}$ of a yard of fabric to make book covers. Each book cover is made from $\frac{1}{8}$ of a yard of fabric. How many book covers can Michael make? Solution: Michael can make 4 book covers.



Example 4:

Represent $\frac{1}{2} \div \frac{2}{3}$ in a problem context and draw a model to show your solution.

Context: A recipe requires $\frac{2}{3}$ of a cup of yogurt. Rachel has $\frac{1}{2}$ of a cup of yogurt from a snack pack. How much of the recipe can Rachel make?

Explanation of Model:

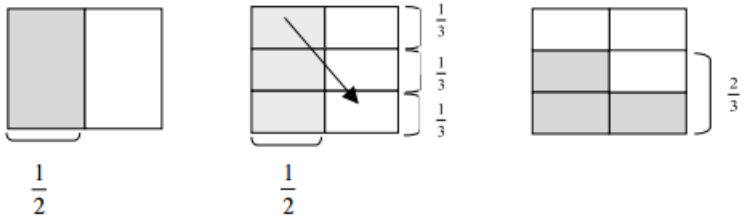
The first model shows $\frac{1}{2}$ cup. The shaded squares in all three models show the $\frac{1}{2}$ cup.

The second model shows $\frac{1}{2}$ cup and also shows $\frac{1}{3}$ cups horizontally.

The third model shows $\frac{1}{2}$ cup moved to fit in only the area shown by $\frac{2}{3}$ of the model.

$\frac{2}{3}$ is the new referent unit (whole) .

3 out of the 4 squares in the $\frac{2}{3}$ portion are shaded. A $\frac{1}{2}$ cup is only $\frac{3}{4}$ of a $\frac{2}{3}$ cup portion, so only ¾ of the recipe can be made.



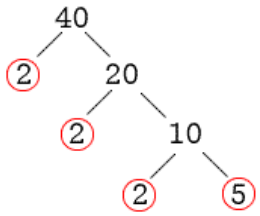
Source: NC DPI 6th Grade Mathematics Unpacked Contents

- Algorithmic approaches to divide multi-digit numbers
- Algorithmic approaches to add, subtract, multiply, and divide decimal numbers
- For division, modeling equivalent numerical expressions will support understanding of moving the decimal point

- Prime factorization

Prime Factor Tree

- Start by dividing the given number by the smallest prime which is 2.
- The factors of the number above are broken down into "branches" as indicated by the line segments.
- We are able to divide **40** and its quotient by the prime number 2 three times which means this prime number will have an exponent of 3 in the factorization.
- The last quotient after repeated division of 2 is a prime number which is 5.
- Upon reaching a prime number as its last quotient in the process, this shows that we are done!

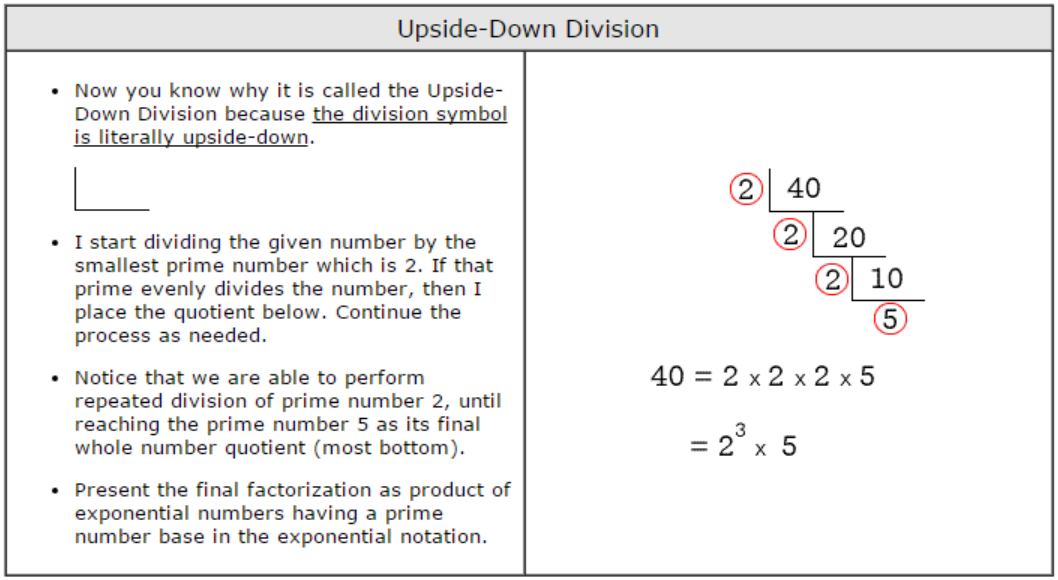


$40 = 2 \times 2 \times 2 \times 5$

$= 2^3 \times 5$

Source: Chili Math

- Upside Down Division (Birthday Cake)



Source: Chili Math

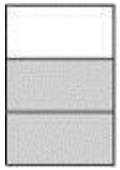
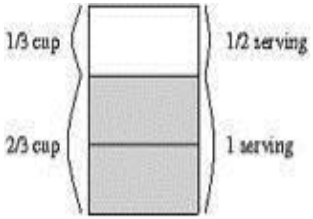
Resources

6.NS.1 - This game allows students to practice the division of fractions. http://www.math-play.com/math-basketball-dividing-fractions-game/math-basketball-dividing-fractions-game.html

6.NS.1 - This performance task requires students to divide fractions, interpret quotients, and support solutions. http://schools.nyc.gov/NR/rdonlyres/946D93E8-E911-4589-871C-97317E227C3C/141874/NYCDOE_G6_Math_SharemyCandy_FINAL.pdf

6.NS.9 - The included activities encourage students to perform operations with fractions and translate between the multiple representations of rational numbers.http://empower.terc.edu/pdf/Using_Benchmarks.pdf

This website provides real-world problems associated with this unit. https://www.illustrativemathematics.org/NS

Sample Formative Assessment Tasks/Questions
6.NS.1: Tonya and Chrissy are trying to understand the following story problem for 1÷2/3. *One serving of rice is* 2/3 *of a cup. I ate 1 cup of rice. How many servings of rice did I eat?* To solve the problem, Tonya and Chrissy draw a diagram divided into three equal pieces, and shade two of those pieces.  Tonya says, "There is one 2/3-cup serving of rice in 1 cup, and there is ⅓ cup of rice left over, so the answer should be 1 ⅓ " Chrissy says, "I heard someone say that the answer is 3/2=1 ½ . Which answer is right?" Is the answer 1 ⅓ or 1 ½ ? Explain your reasoning using the diagram. *Answer* In Tonya's solution of 1 ⅓ , she correctly notices that there is one ⅔ cup serving of rice in 1 cup, and there is cup of rice left over. But she is mixing up the quantities of servings and cups in her answer. The question becomes how many servings is ⅓ cup of rice? The answer is "⅓ cup of rice is 12 of a serving." It would be correct to say, "There is one serving of rice with ⅓ cup of rice left over," but to interpret the quotient 1 ½ the units for the 1 and the units for the ½ must be the same: *There are* 1 ½ *servings in 1 cup of rice if each serving is ⅔ cup.* (Source: Illustrative Mathematics)  Source: Illustrative Mathematics
6.NS.2: Southern Middle School is hosting a football game this Thursday. Band members are selling ads for the game's program. Their goal is to sell $3,462 worth of ads. If the band members sell each ad for $15, determine how many ads they will need to sell to reach their goal. Justify your answer. *Answer* 231 ads - To determine the number of ads to be sold, students should divide the cost of each ad ($15) into the desired sells ($3462). The resulting quotient is 230 with a remainder 12. Since the quotient is 230 with a remainder 12, students need to determine that 231 ads must be sold to reach the goal of $3462 raised.

6.NS.3: Jalyn and her 3 friends bought snacks that cost $2.86, $7.52, $4.38, 2.95, and $3.05. If they share the cost, how much will each student have to pay to divide it evenly?

Answer
$5.23

6.NS.4b: Johnny and Maria like to go to the mall and window shop. Both Johnny and Maria are at the mall at the same time today. Johnny goes to the mall every 5 days while Maria goes to the mall every 6 days. When will Johnny and Maria run into one another again at the mall? (GCF)

Answer
30 days

6.NS.4a,c: Bob scored 24 points in a basketball game while Jim scored 40 points. Write an equivalent numerical expression to find the sum of these two numbers. Justify your thinking.

Answer
8(3 + 5) – To determine this answer the student needs to see that both 24 and 40 have a greatest common factor of 8. The student explains that factoring out the eight in both numbers allows them to write an equivalent expression using the distributive property. By factoring out the 8 you are left with adding 3 and 5 in the parentheses.

6.NS.9: Which form is best to use when comparing rational numbers? Explain your rationale.

Answer
Student answers will vary. Be sure their rationale supports the form they select.

6.NS.9: In this task, students are the head of a basketball team in the NBA. Their three best starters are injured and not available to play in the next game. It is now their task to look at the statistics provided, and decide which five players will start the game.

The Dream Team.pdf

Source: Converting and Ordering Rational Numbers [6th grade]

6.NS.9: Before a game, Jake's batting average was exactly 0.350. That is the decimal representation for number of hits to number of times at bat. During the game, Jake bats 4 times and gets 2 hits. If Jake's batting average after the game is 0.359, how many times had Jake batted before the end of the game? Explain your reasoning.

Answer 64 at bats after the game - To check Jake's batting average after the game with 2 hits in 4 at bats, you can take a row of numbers from the ratio table, add 2 to the first number and 4 to the second, and then evaluate the quotient. For example, if Jake had 7 hits in 20 at bats before the game, then we would have 9 hits in 24 at bats after the game. Since $9/24=3/8=0.375$, this is not the correct value. With 14 hits in 40 at bats, Jake would have 16 hits in 44 at bats for a batting average of $16/44=4/11\approx0.364$ so this is also not correct. With 21 hits in 60 at bats before the game, Jake would have 23 hits in 64 at bats after the game. Since $23/64\approx0.359$, this could be correct. The next value to calculate would be 28 hits in 80 at bats entering the game so 30 hits in 84 at bats after the game: $30/84=5/14\approx0.357$, so this is not correct. The more at bats Jake has before the game, the less impact his 2 hits in 4 at bats have on his overall batting average. The only possibility that fits the given information is that Jake had 21 hits in 60 at bats before the game therefore he had 64 at bats at the end of the game.

Unit Title
Ratios and Rates
Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*
6.RP.1 Interpret the concept of a ratio as the relationship between two quantities, including part to part and part to whole. ○ Ratios compare two quantities ○ Simplify ratios to simplest form **6.RP.2** Investigate relationships between ratios and rates. a. Translate between multiple representations of ratios (i.e., a/b , a : b, a to b, visual models). b. Recognize that a rate is a type of ratio involving two different units. c. Convert from rates to unit rates. ○ When writing a ratio, order of terminology matters ○ Transfer between multiple representations of ratios ○ Understand that operations with ratios are generally performed when the ratio is written in fractional form ○ Exclude complex fractions (i.e. ⅓ / 4) **6.RP.3** Apply the concepts of ratios and rates to solve real-world and mathematical problems. a. Create a table consisting of equivalent ratios and plot the results on the coordinate plane. b. Use multiple representations, including tape diagrams, tables, double number lines, and equations, to find missing values of equivalent ratios. c. Use two tables to compare related ratios. d. Apply concepts of unit rate to solve problems, including unit pricing and constant speed. e. Understand that a percentage is a rate per 100 and use this to solve problems involving wholes, parts, and percentages. f. Solve one-step problems involving ratios and unit rates (e.g., dimensional analysis). ○ Ratios can be used to find missing values in a table ○ Percent is a rate per 100 ○ Ratio reasoning can be used to convert measurement units ○ Include single step dimensional analysis (e.g., converting miles to yards)

New Academic Vocabulary for This Unit
● Rate ● Ratio ● Unit Rate ● Dimensional Analysis (single step)

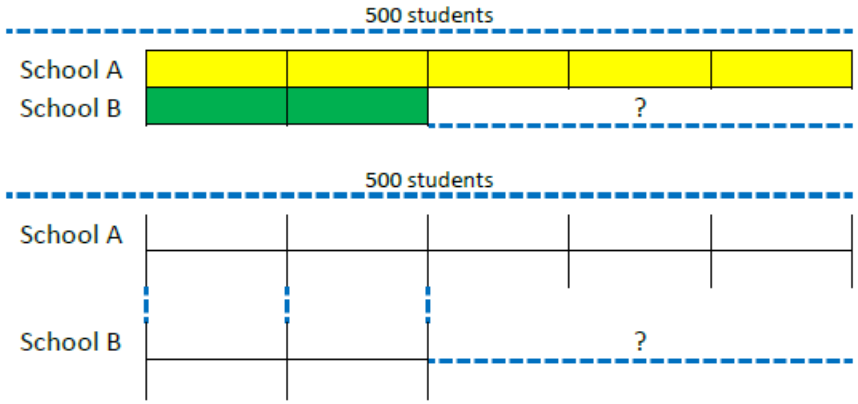
Prior Knowledge Required for this Unit
● Equivalent fractions (3.NSF.2, 4.NSF.1) ● Simplifying fractions (3.NSF.2, 4.NSF.1) ● Multiplication and division of rational numbers (5.NSF.4, 5.NSF.7)
Subsequent Knowledge Related to this Unit
In Grade 5, students developed a conceptual understanding of writing equivalent fractions with unlike and like denominators using visual models. This knowledge will lead students to an understanding of how to simplify ratios and rates down to unit rates. Students will also need to build conceptual knowledge of rates and ratios due to the abstractness of comparing two things within a fraction. The comparison of two quantities with like units will lead students to an analysis of like and unlike units including complex fractions in Grade 7; additionally, students will extend their understanding to geometry by identifying the relationship between diameter and circumference. Work with ratios and rates will also contribute to students connecting this concept to probability, in Grade 7, by finding the number of like outcomes in a comparison with the number of total outcomes. Finding equivalent ratios using a table will lead students to finding a constant of proportionality in Grade 7 and ultimately the constant rate of change (slope) in Grade 8 Functions. In Grade 8, students will also explore transformations of transversals to transformations of similar figures including side lengths and angles.
Relationship Among Standards in this Unit
Standards in this unit will establish an understanding of relationships that exist among quantities of similar and different units.

Potential Instructional Strategies/Lessons

- **Visual Models** - Multiple representations of visual models should be used to show equivalencies among ratios and different missing values.

Tape diagrams

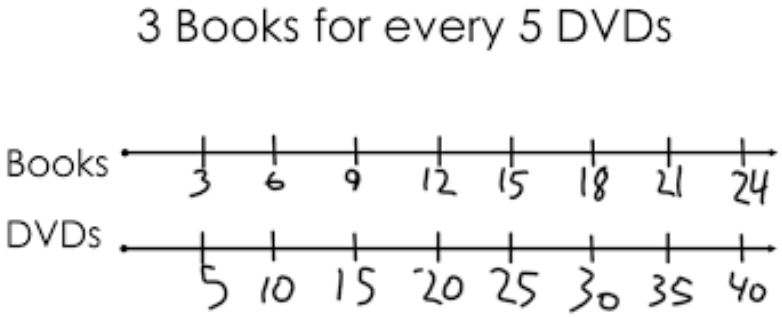
Comparison Model (part-part): School A has 500 students, which is 2 $^{1}/_{2}$ (which is equal to $^{5}/_{2}$) times as many students as School B. How many more students attend School A than School B?



Tables

Girls	Boys
3	2
6	4
9	6
12	8

Double Number Lines



Equations

The equations generated during the ratio unit will be unique in that they follow the form of $y = mx$ or $px = q$ (these notations represent the same thing). The intercept (location the line crosses on the y-axis) will always be zero.

Miles traveled = 20 · number of hours **or** $y = 20x$

- **Dimensional Analysis Tables**

Starting amount	Equal amounts		End Amount
24 inches	1 foot	=	feet
	12 inches		
24 ~~inches~~	1 foot	=	2 feet
	12 ~~inches~~		

Source: Chemistry Land

Resources

6.RP.2 - This video discusses the concepts of ratios, rates, and unit rates: http://mathantics.com/index.php/section/lesson/ratiosandrates

6.RP.2 - This site allows students to practice simplifying ratios and identifying equivalent ratios while racing a dirt bike around a track. http://www.mathplayground.com/ASB_Index.html

6.RP. 3 - This task requires students to use proportional reasoning to compare the size of a typical hamburger with that of the largest hamburger ever made. http://www.yummymath.com/2015/big-burger-2/

6.RP.1-3 - This site includes a full lesson plan, activity instructions, and accompanying worksheets for ratio/rates competition stations. http://www.uen.org/Lessonplan/preview.cgi?LPid=23491

Conceptual Foundations for Ratios and Proportions - http://elemmath.jordandistrict.org/files/2012/06/CF1.docx

Sample Formative Assessment Tasks/Questions

6.RP.1: Pianos and pipe organs contain keyboards, a portion of which is shown below.

a. What is the ratio of black keys to white keys in the picture above?
b. If the pattern shown continues, how many black keys appear on a portable keyboard with 35 white keys?
c. If the pattern shown continues, how many black keys appear on a pipe organ with a total of 240 keys?

Answers

a. 5:7

b.

black keys	5	10	15	20	25
white keys	7	14	21	28	35

c.

black keys	5	10	100
total keys (black keys + white keys)	12	24	240

Source: The New York City Department of Education

6.RP.2: There are 12 boys and 16 girls in a classroom. Which represents the simplified ratio of girls to students in the classroom?
a. 3 to 4
b. 4 to 3
c. 4 to 7
d. 7 to 4

Answer
C, to determine the answer to this question the students need to read the question and work left to right with setting up the ratio.
The numerator of the ratio is 16 for girls and the denominator can be calculated by adding the number of boys and girls together to get 28.
This gives me the ration 16/28. I can simplify this down to 4/7 by dividing by the greatest common factor of 4.

6.RP.3: Dianne went for a ride on her new scooter. She traveled 450 meters in 36 seconds. Which statements are true? Select all that apply.
a. She traveled 12.5 meters every second.
b. Every 75 seconds she traveled 6 meters.
c. Every 8 seconds she traveled 100 meters.
d. She traveled 1 meter in 0.8 second.
e. Every 24 seconds, she traveled 300 meters.

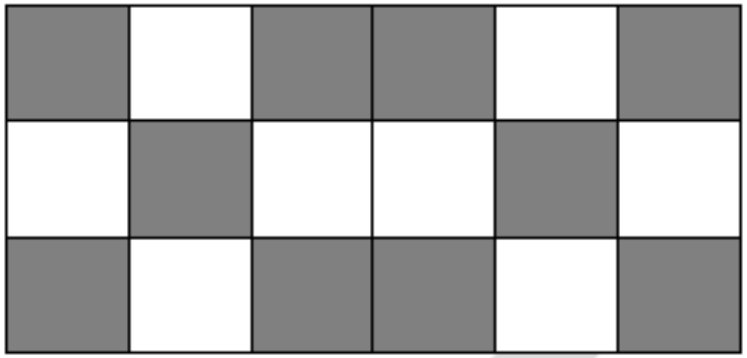
Answer
A, C, E - To determine the answer to this question, students must determine the unit rate that represents Dianne's speed. Since she is traveling at a rate of 12.5 meters for every one second, answer choice A is correct. Answer choice B is incorrect because it did not maintain the correct order in the ratio; it would be 75 meters for every 6 seconds. Answer choice C is correct because 12.5 meters multiplied by 8 is 100 meters, and 1 second multiplied by 8 is 8 seconds. By multiplying each part of the rate by the same factor we maintain an equivalent ratio. Answer choice D is incorrect because the rate was simplified incorrectly. A correct statement would read that, "She traveled 1 meter in 0.08 seconds." Answer choice E is correct because 12.5 meters multiplied by 24 is 300 meters, and 1 second multiplied by 24 is 24 seconds.

6.RP.1: This task requires students to use their knowledge of elapsed time and proportional reasoning to answer questions about record setting sporting events.

Longest_NHL_Matches_Ever.pdf

Source: Yummy Math

6.RP.1 and 6.RP.2: The new floor in the school cafeteria is going to be constructed of square tiles that are either gray or white and in the pattern that appears below:



Part A: What is the ratio of gray tiles to white tiles?
Answer _____
Part B: What is the ratio of white tiles to the total number of tiles in the pattern?
Answer _____
Part C: If the total cost of the white tiles is $12, what is the unit cost per white tile?
Answer $_____

Answer
Part A: 10 to 8, 5:4, or other equivalent ratio. The correct answer is a ratio of 10 gray tiles to 8 white tiles, or simplified, the ratio will be 5 gray tiles to 4 white tiles.
Part B: 8 to 18, 4:9, or other equivalent ratio. The correct answer is a ratio of 8 white tiles to 18 total tiles, or simplified, the ratio will be 4 white tiles to 9 tiles, in total.
Part C: $1.50 per white tile. Counting the tiles by color in the pattern above, it is found that there are 8 white tiles. If 8 white tiles cost $12, then the cost per white tile is $1.50.

Unit Title
Graphing and Rational Numbers
Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*
6.NS.5 Understand that the positive and negative representations of a number are opposites in direction and value. Use integers to represent quantities in real-world situations and explain the meaning of zero in each situation. ○ Include temperature, elevation, credits/debits ○ Include vertical and horizontal number lines **6.NS.6** Extend the understanding of the number line to include all rational numbers and apply this concept to the coordinate plane. a. Understand the concept of opposite numbers, including zero, and their relative locations on the number line. b. Understand that the signs of the coordinates in ordered pairs indicate their location on an axis or in a quadrant on the coordinate plane. c. Recognize when ordered pairs are reflections of each other on the coordinate plane across one axis, both axes, or the origin. d. Plot rational numbers on number lines and ordered pairs on coordinate planes. ○ Include vertical and horizontal number lines, plot all rational numbers ○ Understand the effects of reflections on the ordered pairs (e.g., the ordered pair (2, 3) reflected about the y-axis becomes (-2, 3)) **6.NS.7** Understand and apply the concepts of comparing, ordering, and finding absolute value to rational numbers. a. Interpret statements using equal to (=) and not equal to (≠). b. Interpret statements using less than (<), greater than (>), and equal to (=) as relative locations on the number line. c. Use concepts of equality and inequality to write and to explain real-world and mathematical situations. d. Understand that absolute value represents a number's distance from zero on the number line and use the absolute value of a rational number to represent real-world situations. e. Recognize the difference between comparing absolute values and ordering rational numbers. For negative rational numbers, understand that as the absolute value increases, the value of the negative number decreases. ○ Limit inequalities to simple statements of comparison (e.g., comparing a loss of 5 yards to a loss of 3 yards) ○ Limit comparing values with $<, >, =, \neq$ ○ Recognize absolute value is a distance and not the opposite of the number ○ Understand that distance is always positive (it is the direction that changes) **6.NS.8** Extend knowledge of the coordinate plane to solve real-world and mathematical problems involving rational numbers. a. Plot points in all four quadrants to represent the problem. b. Find the distance between two points when ordered pairs have the same x-coordinates or same y-coordinates.

c. Relate finding the distance between two points in a coordinate plane to absolute value using a number line.
- Recognize the x-axis as a horizontal number line and the y-axis as a vertical number line
- Plot points that involve all rational numbers
- Limit distance between points to horizontal distances (having the same x-coordinates) or vertical distances (having the same y-coordinates)
- Recognize the final value of a distance between two points results in a positive value

New Academic Vocabulary for This Unit

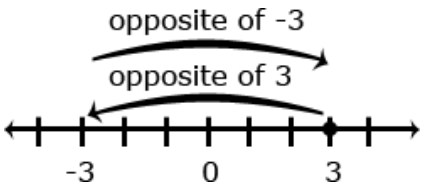
- Integers
- Quadrant
- Opposite
- Zero pair
- Additive inverse
- Absolute value
- Inequality (including $\neq$)

Prior Knowledge Required for this Unit
● Coordinate system (5.G.1, 5.G.2) ● Comparison (3.NSBT.5, 4.NSF.2, 5.NSBT.3)
Subsequent Knowledge Related to this Unit
In Grades 3, 4, and 5, students compare whole numbers, fractions, and decimals using <, >, or =. In Grade 6, students will compare all rational numbers using <, >, =, or ≠.In Grade 7, students will extend knowledge of inequalities to include $\leq$ and $\geq$.In Grade 8 and high school courses, students will continue to compare numbers to include irrational numbers, complex numbers, and imaginary numbers. In Grade 5, students are introduced to the coordinate system and graphing in Quadrant I of the coordinate plane. This knowledge will lead students to an understanding of how to plot points in all four quadrants. With the lack of graphing in Grade 7 on the coordinate plane, students need to build a strong foundation of the coordinate system (i.e., x- and y-axes, origin, ordered pairs, four quadrants, graphing x-values before y- values). Content in this unit is linked to constant of proportionality in Grade 7, transformational geometry, graphing linear equations and systems of linear equations, and finding the distance between two points using the Pythagorean Theorem in Grade 8. In High School mathematics courses, students will extend this knowledge to graph nonlinear functions as well as finding the distance between two points using the distance formula and the midpoint formula. Students will also use the knowledge of graphing points to include imaginary and complex values.
Relationship Among Standards in this Unit
Standards in this unit will establish an understanding of relationships that exist among positive and negative representations of rational numbers.

Potential Instructional Strategies/Lessons

- **Horizontal and Vertical Number Lines**

For example, – 3 could be read as "the opposite of 3" which would be negative 3. Zero is its own opposite.



Example 1:

What is the opposite of $2\frac{1}{2}$? Explain your answer.

Answer
$-2\frac{1}{2}$ because it is the same distance from 0 on the opposite side.
Students worked with Quadrant I in elementary school. As the x-axis and y-axis are extending to include negatives, students begin to with the Cartesian Coordinate system. Students recognize the point where the x-axis and y-axis intersect as the origin. Students identify the four quadrants and are able to identify the quadrant for an ordered pair based on the signs of the coordinates. For example, students recognize that in Quadrant II, the signs of all ordered pairs would be (-, +)

Students understand the relationship between two ordered pairs differing only by signs as reflections across one or both axes. For example, in the ordered pairs (-2, 4) and (-2, -4), the y-coordinates differ only by signs, which represents a reflection across the x-axis. A change is the x-coordinates from (-2, 4) to (2, 4), represents a reflection across the y-axis. When the signs of both coordinates change, [(2, -4) changes to (-2, 4)], the ordered pair has been reflected across both axes.

Source: NC DPI 6th Grade Mathematics Unpacked Contents

Example 2:
Graph the following points in the correct quadrant of the coordinate plane. If the point is reflected across the x-axis, what are the coordinates of the reflected points? What similarities are between coordinates of the original point and the reflected point?

$(\frac{1}{2}, -3\frac{1}{2})$ $(-\frac{1}{2}, -3)$

Answer
The coordinates of the reflected points would be $(\frac{1}{2}, 3\frac{1}{2})$ and $(-\frac{1}{2}, 3)$. Note that the y-coordinates are opposites.

Example 3:
Students place the following numbers would be on a number line: -4.5, 2, 3.2, $-3\frac{3}{5}$, 0.2, -2, $\frac{11}{2}$. Based on number line placement, numbers can be placed in order.

Answer
The numbers in order from least to greatest are: -4.5, $-3\frac{3}{5}$, -2, 0.2, 2, 3.2, $\frac{11}{2}$. Students place each of these numbers on a number line to justify this order.

Source: NC DPI 6th Grade Mathematics Unpacked Contents

Resources

Online Graphing Calculator: https://www.desmos.com/calculator

6.NS.7 - This jeopardy game allows students to compare rational numbers http://www.math-play.com/Comparing-Rational-Numbers/comparing-rational-numbers.html

6.NS.6 - This game allows students to compare integers.
http://www.xpmath.com/forums/arcade.php?do=play&gameid=61

6.NS.6 - This activity allows students to understand the concept of opposite numbers and absolute value. https://www.ixl.com/math/grade-7/absolute-value-and-opposite-integers

6.NS.8 - This activity allows students to locate points on the coordinate plane. http://www.math-play.com/Coordinate%20Plane%20Game/Graphing-points-in-the-coordinate-plane.html

Sample Formative Assessment Tasks/Questions
6.NS.5: Denver, CO is called the "Mile High City" because its elevation is 5280 feet above sea level. Someone tells you that the elevation of Death Valley, CA is -282 feet. a. Is Death Valley located above or below sea level? Explain. b. How many feet higher is Denver than Death Valley? *Answer* a. Because the elevation is negative, Death Valley is located below sea level. Places that are above sea level have positive elevations. A vertical number line can be used to display this understanding. b. Death Valley is 282 feet below sea level, so it takes 282 feet to get to sea level. Denver is 5280 feet above sea level, so we add that distance to the 282 feet we already have. 282 + 5280 = 5562. Denver is 5562 feet higher above Death Valley. Source: Illustrative Mathematics

6.NS.6:

a. For each set of points below, draw and label a set of coordinate axes and plot the points:

   i. (2, 3), (1, −5), (−3, 0), (-4, $3\frac{1}{2}$)
   ii. (70, 70), (0, 0), (−20, −35), (55, -45)
   iii. ($\frac{1}{8}$, $\frac{7}{8}$), ($\frac{-5}{2}$, 0.25), ($-1\frac{1}{4}$, $-\frac{1}{8}$), (0.5, $-\frac{3}{4}$)

b. Would it be reasonable to use the same scale for plotting each set of points from part (a)? Explain your reasoning.

c. How do the points impact your decision of scale for the axes?

Answer

a. *Click the link beside the set of ordered pairs.*

   i. (2, 3), (1, −5), (−3, 0), (-4, $3\frac{1}{2}$) LINK TO GRAPH
   ii. (70, 70), (0, 0), (−20, −35), (55, -45) LINK TO GRAPH
   iii. ($\frac{1}{8}$, $\frac{7}{8}$), ($\frac{-5}{2}$, 0.25), ($-1\frac{1}{4}$, $-\frac{1}{8}$), (0.5, $-\frac{3}{4}$) LINK TO GRAPH

b. No, it would not be reasonable to use the same scale for the axes. The range between values within the sets is too large.

c. You would get a more accurate representation of the relationship between points. For example, you could use a scale of: 1 for (i); 10 for (ii); $\frac{1}{4}$ for (iii).

6.NS.7: The record low temperatures (in Celsius) in Gaffney, SC for one week include: Monday ($5^{\circ}C$), Tuesday ($-1^{\circ}C$), Wednesday ($-6^{\circ}C$), Thursday ($-2^{\circ}C$), Friday ($3^{\circ}C$), Saturday ($7^{\circ}C$), and Sunday ($0^{\circ}C$).

a. List these temperatures in order from least to greatest.
b. On a spring day in Portland, OR, the low temperature is 13 degrees below zero (in Celsius), and the low temperature in Nashville, TN was 4 degrees below zero (in Celsius). Daniela wrote Nashville was colder because -4 < -13. Is Daniela correct? Explain your answer.
c. Antarctica has the coldest temperature ever recorded: -89 degrees Celsius. The average temperature on Jupiter is approximately -145 degrees Celsius. Which is warmer, the average temperature on Jupiter or the coldest temperature on Earth? Write an inequality to support your answer.

Answer

a. $-6^{\circ}C, -2^{\circ}C, 0^{\circ}C, 3^{\circ}C, 5^{\circ}C, 7^{\circ}C$ (Students may want to plot the temperatures on a number line to assist with ordering values.)
b. Daniela is incorrect. $13^{\circ}C$ is less than $-4^{\circ}C$
c. -145 < -89; therefore, the coldest temperature on Earth is warmer than the average temperature of Jupiter.

6.NS.8: Ms. Johnson is making a map of a local amusement park by plotting locations on a coordinate plane, where one unit represents 100 feet, and the origin represents the ticket stand. Ms. Johnson plots the Rapid Racer Roller Coaster at (-6, -3) and the Wet and Wild Water Coaster at (-10, -3).

What is the distance between the two rides on the coordinate plane? What does this distance represent in real life?

Answer

The rides are four units away from one another on the coordinate plane. This is equivalent to 400 feet apart in the actual amusement park.

Unit Title
Expressions
Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*
6.EEI.1 Write and evaluate numerical expressions involving whole-number exponents and positive rational number bases using the Order of Operations. ○ Perform arithmetic operations, including those involving whole-number exponents, using order of operations, including expressions with and without parentheses. **6.EEI.2** Extend the concepts of numerical expressions to algebraic expressions involving positive rational numbers. a. Translate between algebraic expressions and verbal phrases that include variables. b. Investigate and identify parts of algebraic expressions using mathematical terminology, including term, coefficient, constant, and factor. c. Evaluate real-world and algebraic expressions for specific values using the Order of Operations. Grouping symbols should be limited to parentheses, braces, and brackets. Exponents should be limited to whole-numbers. ○ Read, write, and evaluate expressions in which letters (variables) stand for numbers. ○ Distinguish the difference between an algebraic and numerical expression. ○ Identify parts of an expression using mathematical terms. ○ View one or more parts of an expression as a single entity. ○ Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. **6.EEI.3** Apply mathematical properties (e.g., commutative, associative, distributive) to generate equivalent expressions. ○ Generate equivalent numeric and algebraic expressions. ○ Understand the following mathematical properties of addition: commutative, associative, additive identity, and additive inverse ○ Understand the following mathematical properties of multiplication: commutative, associative, multiplicative identity, multiplicative inverse, and distributive **6.EEI.4** Apply mathematical properties (e.g., commutative, associative, distributive) to justify that two expressions are equivalent. ○ Justify equivalency of numeric and algebraic expressions.

New Academic Vocabulary for This Unit
● Base ● Coefficient ● Constant ● Defining the variable ● Equivalent expressions ● Exponent ● Grouping symbols ● Like terms ● Substitute ● Term

Prior Knowledge Required for this Unit
● Order of Operations (5.ATO.1) ● Translate numerical expressions (5.ATO.2) ● Represent the problem using an equation with a variable as the unknown quantity (4.ATO.3)
Subsequent Knowledge Related to this Unit
In subsequent math courses, students will evaluate and simplify more complex numerical and algebraic expressions to include all rational numbers and their operations. In Grade 7, students will extend the use of grouping symbols to include fraction bars and solve linear equations using the distributive property. In Grade 8, students will use integer exponents and explore the concepts of perfect squares, perfect cubes, square roots, and cube roots. In High School courses, students will work with more complex expressions.
Relationship Among Standards in this Unit
Standards in this unit will establish an understanding of relationships that exist among numeric and algebraic expressions and mathematical properties.

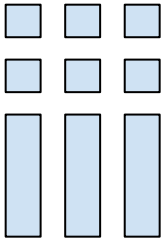
Potential Instructional Strategies/Lessons

- Visual Models

Example 1: The expression 10x + 15 can represent the area of the figure below. Students find the greatest common factor (5) to represent the width and then use the distributive property to find the length (2x + 3). The factors (dimensions) of this figure would be 5(2x + 3).

10x	15

Example 2: Students use their understanding of multiplication to interpret 3 (2 + x) as 3 groups of (2 + x). They use a model to represent x, and make an array to show the meaning of 3(2 + x). They can explain why it makes sense that 3(2 + x) is equal to 6 + 3x. An array with 3 columns and x + 2 in each column:



Source: NC DPI 6th Grade Mathematics Unpacked Contents

Resources
6.EEI.1, 6.EEI.2, 6.EEI.3, 6.EEI.4 - This module set provides tasks, guided practice, collaborative work, homework, and assessments for each of the standards listed. http://www.nps.k12.nj.us/IRC/site/handlers/Grade6ExpressionsandEquationsTeacherModule-moduleinstanceid=12279&dataid=9560&FileName=Grade6ExpressionsandEquationsTeacherModule.pdf.pdf 6.EEI.2, 6.EEI.3, 6.EEI.4 - This performance task has students consider algebraic expressions and mathematical properties as they solve problems involving a grocery list and quilt. http://schools.nyc.gov/NR/rdonlyres/F7DD52E7-FD7E-44DC-B099-90301CAC9025/140802/NYCDOE_G6_Math_GroceryShopping_FINAL1.pdf%20Janel
Sample Formative Assessment Tasks/Questions
6.EEI.1 and 6.EEI.2: Write an expression that is equivalent to 64 using all of the following numbers and symbols once in the expression. **7, 7, 7, ^{2}(exponent of 2),+, ÷, ( )** *Answer* $(7 \div 7 + 7)^2$; The first thing that needed to be recognize by the student is that they must use all of the numbers and symbols when making their expression. The order of operations inside the parentheses should have the student doing the division between the first 2 sevens first which gives a quotient of 1, next you would need to add 1 and 7 to get a total of 8. After the parentheses are done the squaring of 8 will give you and answer of 64 for the expression using all of the necessary symbols.

6.EEI.2: For the school dance only three members from the student government sold tickets. James sold ten more tickets than Bonnie, and Bonnie sold twice as many tickets as Mike.

a. Write an algebraic expression to represent how many tickets all three members sold.
b. A total of 295 tickets were sold for the dance. Using the information above, write an algebraic equation to calculate how many tickets James, Bonnie, and Mike sold individually.
c. Since the party and refreshments for the dance were donated, one hundred percent of all ticket sales from the dance are considered profit. If 295 students attended the dance at a ticket cost of $6, how much money did the student government make from the dance?

Answer

a. Let Mike = x, Bonnie = $2x$, James = $2x+10$; you must assign a variable to each student involved in the question. Mike is assigned the value of x because his expression is not based upon the actions of any other students. After determining how much Mike is worth you can write the expression for both Bonnie and James because they depend on the value of Mike.

b.

$5x + 10=295$	Step 1: Simplify both sides of the equation.
$5x +10 - 10 = 295 -10$	Step 2: Subtract 10 from both sides.
$5x = 285$	Step 3: Divide both sides by 5.
$x = 57$	

Substitute value of x into each express Mike sold 57 tickets, Bonnie sold 114 tickets, and James sold 124 tickets.

c. $295 \times \$6 = \$1,770.00$
The student government made $1,770.00.

6.EEI.3: Which of the following expressions is NOT equivalent to the expression $2x+6$? Why? If you were to correct the ones that are NOT equivalent what should it be?

a. $x + x + 2 + 2 + 2$
b. $2(x + 3)$
c. $2x + 2^3$
d. $4 + x + 2 + x$

Answer

a. $x + x + 2 + 2 + 2 = 2x + 6$
b. $2(x + 3) = 2x + 6$
c. $2x + 2^3 = 2x + 8$
d. $4 + x + 2 + x = 2x + 6$

The equivalent expressions are a, b, and d. When you combine like terms, they generate

6.EEI.3 and 6.EEI.4: Write the expression 16 + 14 as a product of two factors. Use properties of multiplication and addition to justify your answer.

16 + 14 The GCF = 2	Step 1: Find the greatest common factor (GCF) of 16 and 14.
2(8 + 7)	Step 2: Factor that number out using the distributive property
2(7 + 8)	Step 3: Commutative property of addition (OPTIONAL)